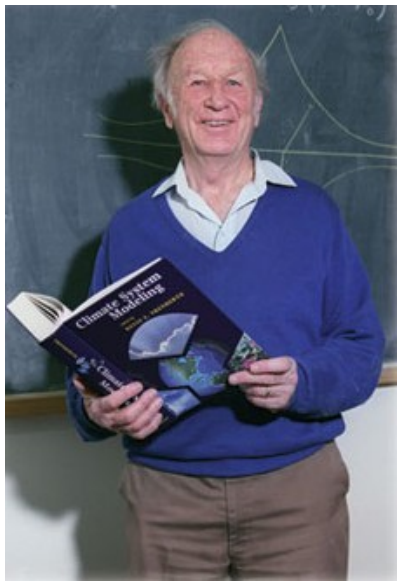
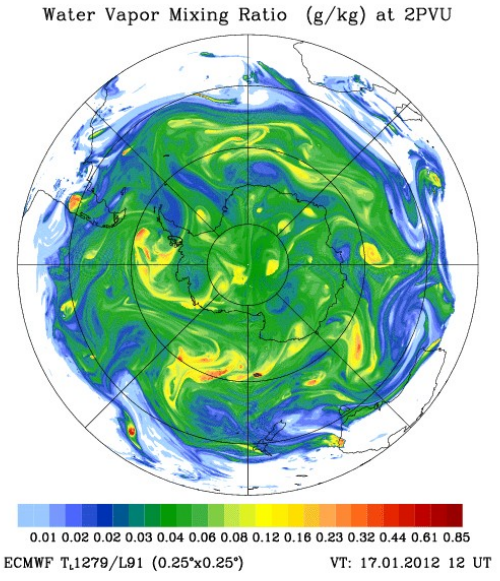
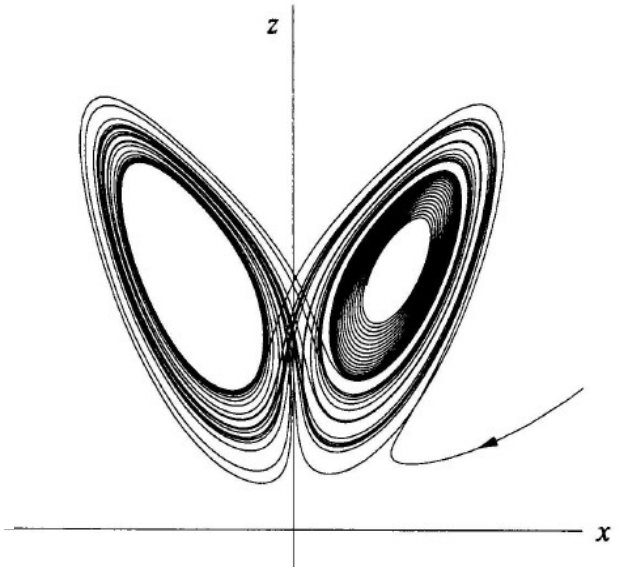
Edward Lorenz: Predictability

Master Literature Seminar, speaker: Josef Schröttle





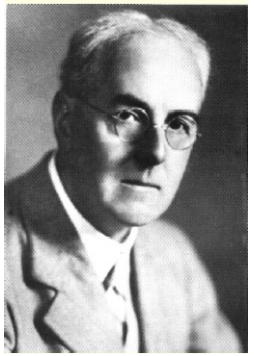


Edward Lorenz in 1994, Northern Hemisphere, Lorenz Attractor

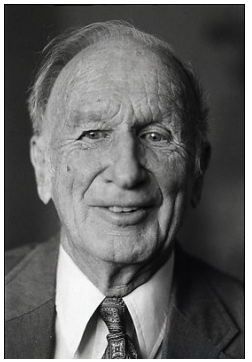
I) Lorenz, E.N.: Deterministic Nonperiodic Flow (JAS, 1963)

II) Lorenz, E.N.: Predictability – a problem partly solved (1996)

Beyond Newton, 20th century ...



Richardson
1881-1954



Lorenz
1917-2008

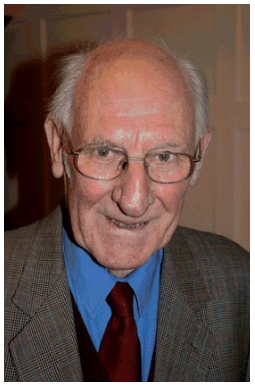
Mandelbrot
1924-2010

Schumann et al.

Astrophysics — **Meteorology** — **Chaos** — **Fluid Dynamics**

Poincaré
1854-1912

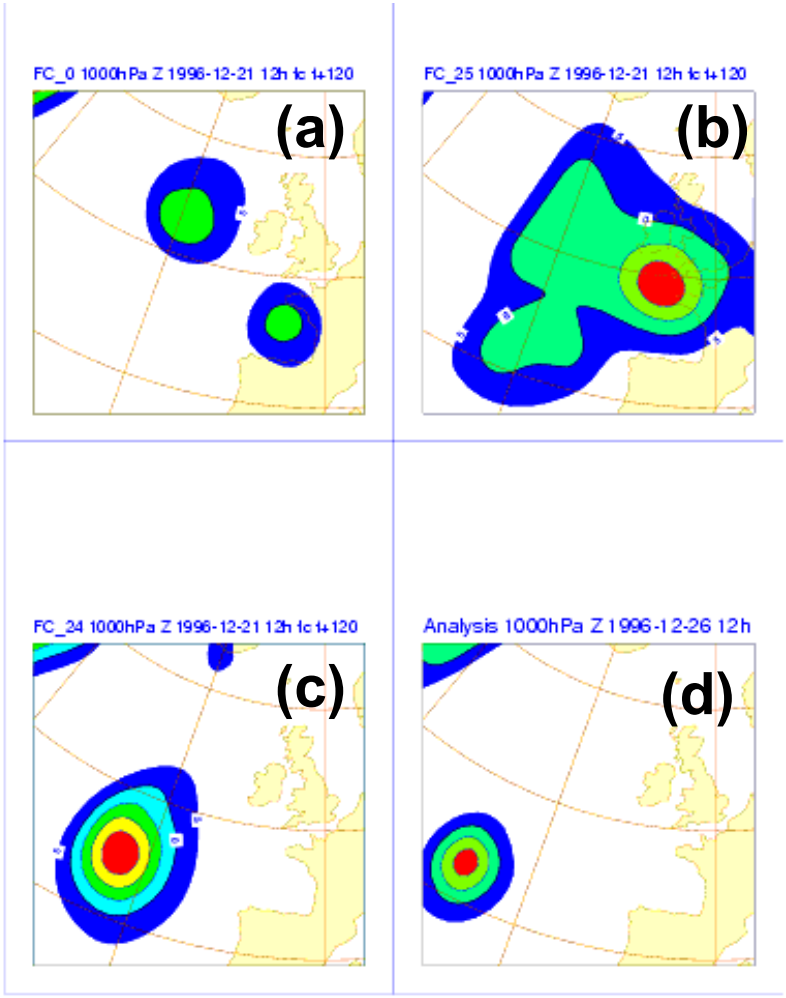
Hide



Feigenbaum

Read et al.

Nowadays: Ensemble forecasts



'5 days ECMWF ensemble forecast', R. Buizza (2000):

Here, an **ensemble forecast** was performed for three different scenarios (a, b and c). Shown is the geopotential height field at 1000 hPa. The initial conditions were **slightly perturbed**. After the 5 day integration period, all three forecasts significantly **diverged**. Closest to the validation d is forecast c, where a **cyclone evolves** west of France over the Atlantic, less intense than predicted.

Hydrodynamical System

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \ldots = -\frac{1}{\rho}\nabla p - \nu \Delta u + F$$

Convection rolls in the atmosphere

CHARACTERISTICS

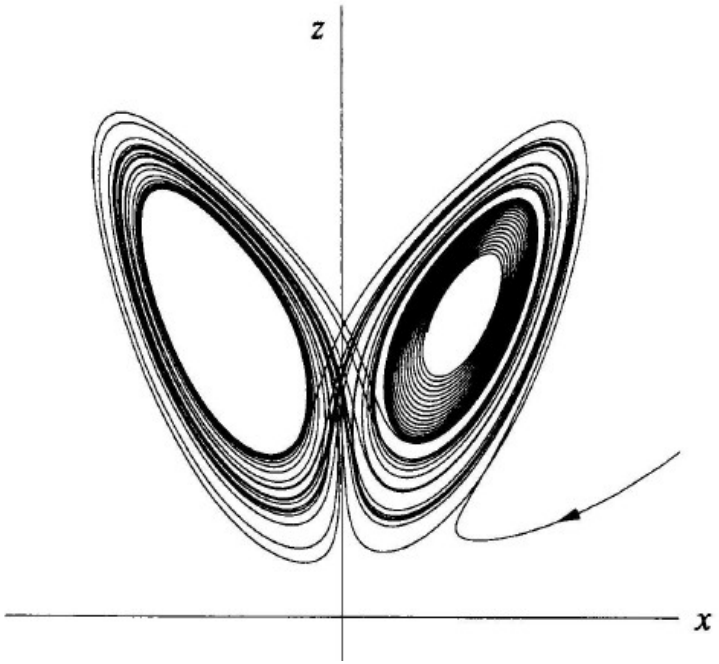
- Viscous **dissipation**

  Thermal **diffusion**

  Energy dissipation

  Nonconservative in terms of Hamiltonian mechanics
- **External** deterministic **forcing** (e.g. buoyancy, coriolis, centrifugal)
- **Nonlinear**, e.g. advection

Lorenz 1963 Model



$$\dot{X} = -\sigma X + \sigma Y$$

$$\dot{Y} = -X Z + \gamma X - Y$$

$$\dot{Z} = X Y - \beta Z$$

PHASE SPACE IN THE LORENZ SYSTEM

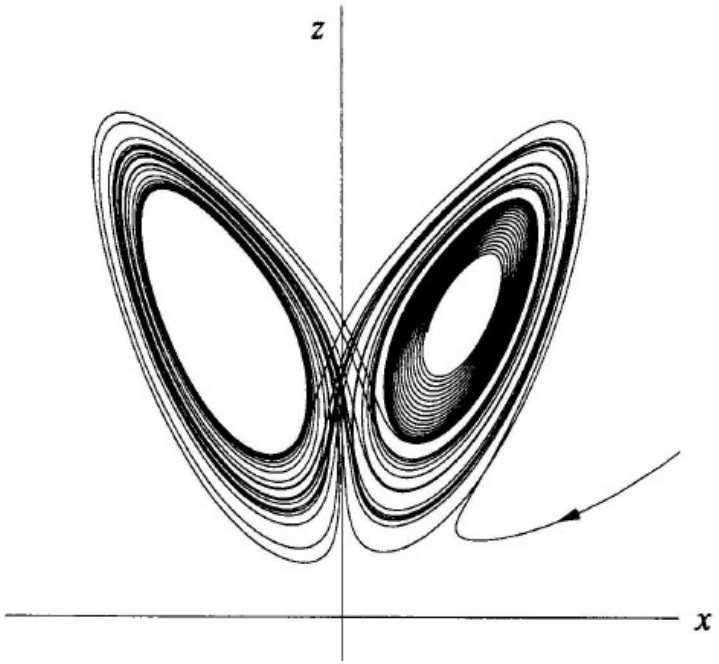
X **Intensity of convective** motion

Y **Temperature difference** of ascending/descending particles

Z **Distortion** of vertical **temperature profile**

σ **Prandtl number**

Lorenz 1963 Model



$$\dot{X} = -\sigma X + \sigma Y$$

$$\dot{Y} = -X Z + \gamma X - Y$$

$$\dot{Z} = X Y - \beta Z$$

PROPERTIES

- Nonlinear
- Dissipative, Nonconservative: $\nabla \cdot (\dot{X}, \dot{Y}, \dot{Z}) = -\sigma - 1 - b \neq 0$
- Exterior forcing due to buoyancy, temperature difference Y
- Chaotic: **Lyapunov Exponent $\lambda_1 > 0$** exists

Lorenz 1963 Model – so what?

(a)

(b)

(c)

SOLUTION

Two **fixed points** as states of steady convection:

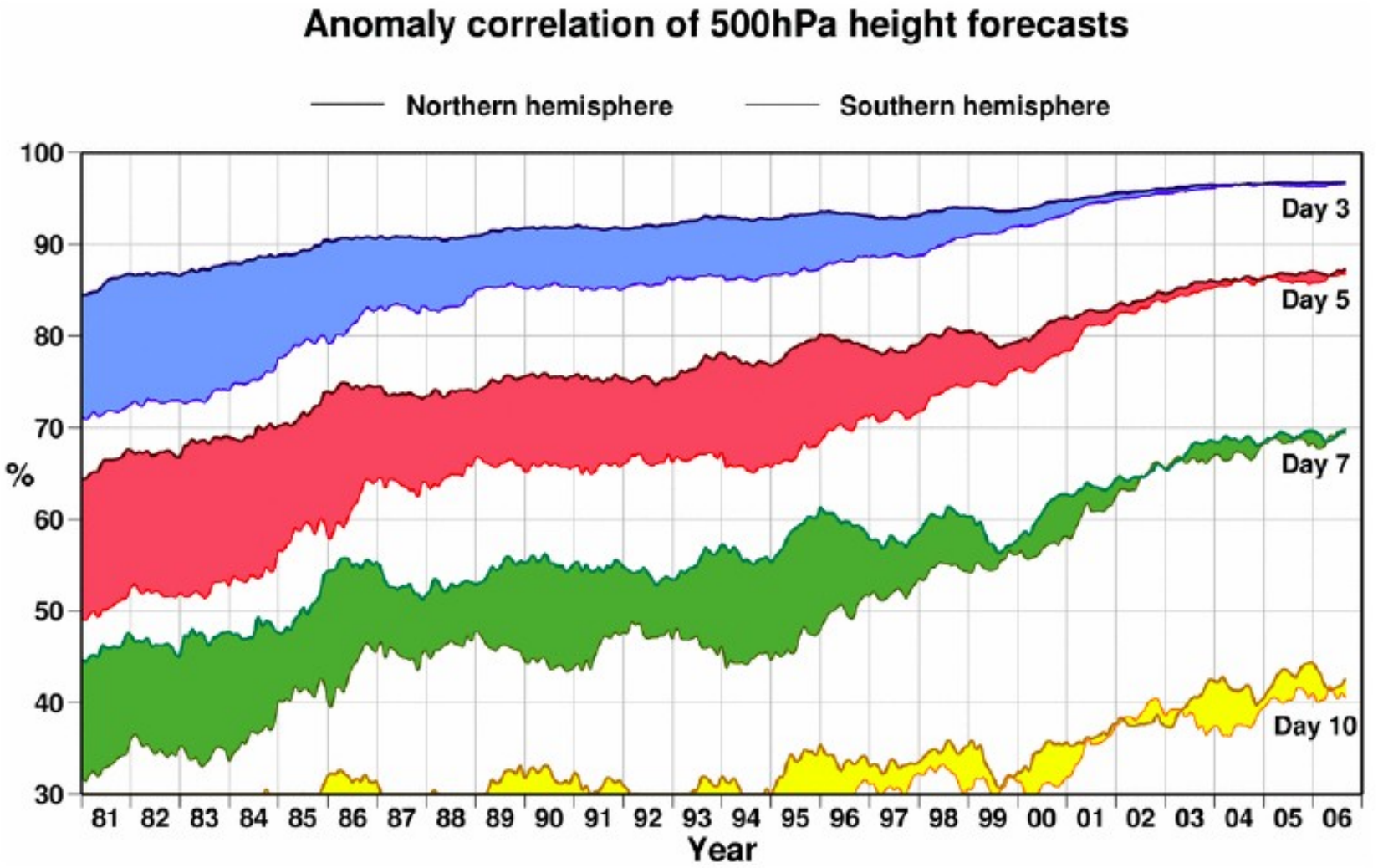
$(6\sqrt{2}, 6\sqrt{2}, 27)$ and $(-6\sqrt{2}, -6\sqrt{2}, 27)$

Here, three experiments show the propagation of initial states in the phase space of the lorenz system to a later stage. Considering the left and right side of the **attractor** as

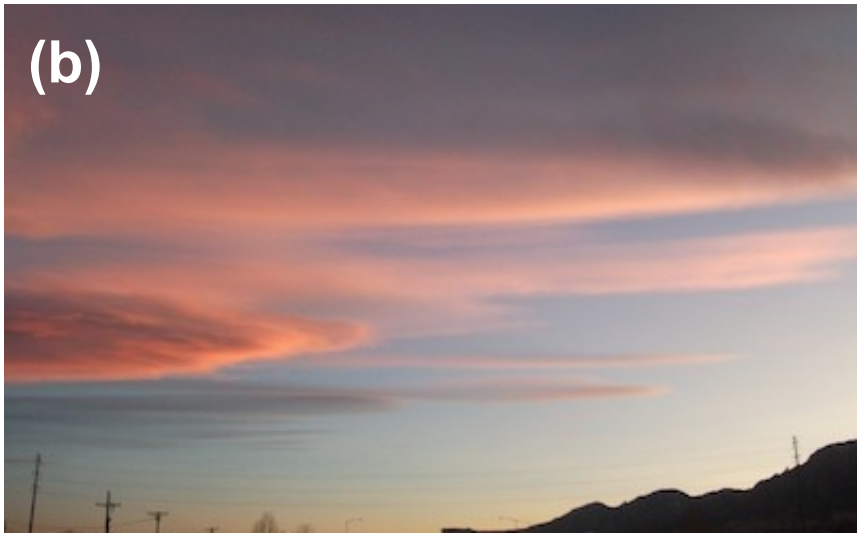
a) a **forecast** is **possible**
b) predictability is **limited**
c) a prediction is **not possible**

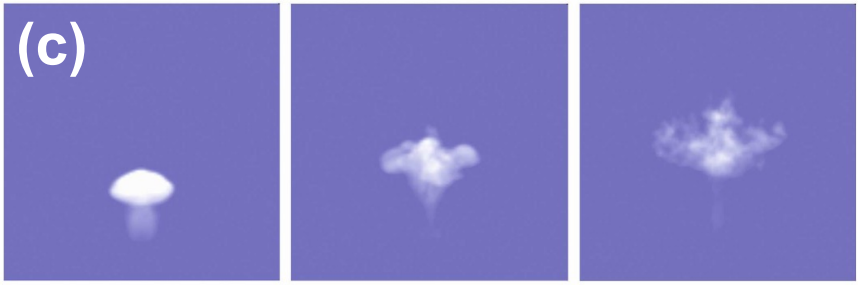
MOVIE STROGATZ

Weather Prediction



Multiscale Phenomena





a) ***Cumulus clouds*** *in Colorado National Park*

b) ***Mountain Wave Clouds*** *in Boulder, Colorado*

c) ***Cumulus cloud*** *in numerical simulation: Craig & Dörnbrack (JAS, 2008)*

Lorenz 1995 Model, System I

$$\frac{dX_k}{dt} = -X_{k-2}X_{k-1} + X_{k-1}X_{k+1} - X_k + F$$

SCILAB EXPERIMENT

PROPERTIES

- Nonlinear in advection terms, dissipative, **cyclic**
- Two advection terms conserve total energy: $\left(X_1^2 + X_2^2 + \ldots X_K^2\right)/2$
- Exterior forcing due to buoyancy, temperature difference Y
- Error **doubling time** 2.1 days

Lorenz 1995 Model, System II

I) Large scale property of the atmosphere e.g. **stratification**

$$\frac{dX_k}{dt} = -X_{k-2}X_{k-1} + X_{k-1}X_{k+1} - X_k - (hc/b)\sum_j Y_{j,k}$$

II) Convective scale quantity e.g. **cloud**

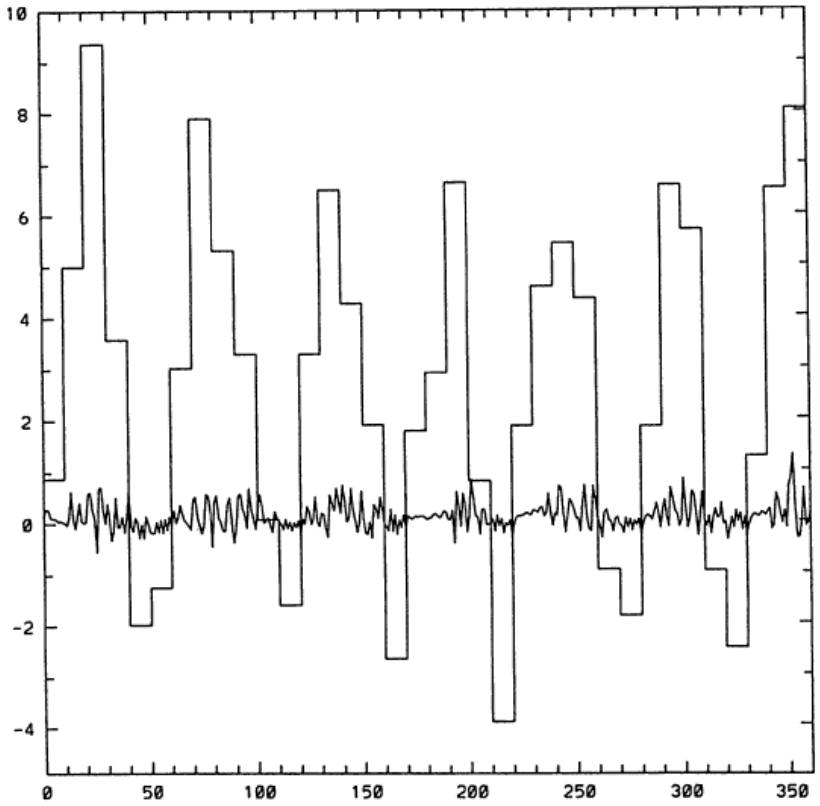
$$\frac{dY_{j,k}}{dt} = -cbY_{j+1,k}\left(Y_{j+2,k} - Y_{j-1,k}\right) - cY_{j,k} + (hc/b)X_k$$

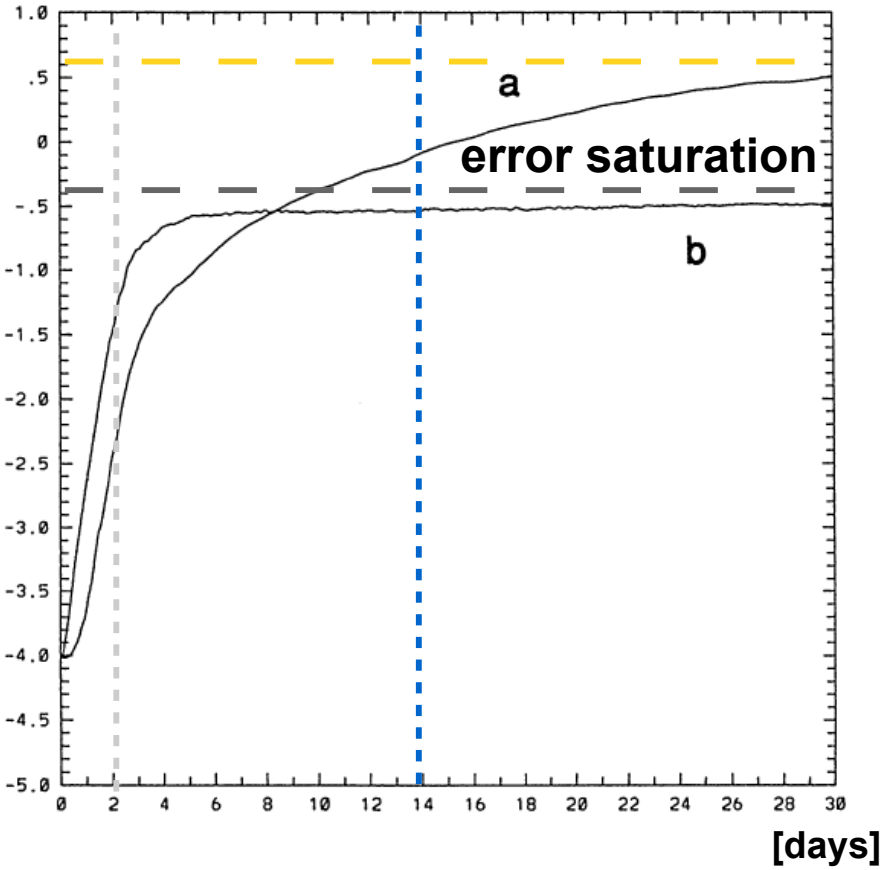
PROPERTIES

- Nonlinear in advection terms, dissipative
- Two advection terms conserve total energy: $\left(X_1^2 + X_2^2 + \ldots X_K^2\right)/2$
- Exterior forcing, here: coupling between the scales, **cyclic**

  with a total of *JK* convective scale equations, *K = 36 and J = 10*

Lorenz 1995 Model, System II





Convective scale *motion* ***$Y_{j,k}$*** *and* ***large scale profile*** *of* ***X_k***

To determine ***error doubling****:* *$\log_{10} E$ of X_k (a) and $Y_{j,k}$ (b)*

EXPERIMENT HEINER

Weather Prediction – happy End?

Orrel (JAS 2003)

- **Refining parametrizations** strong impact on **climate modeling**
- Introducing **stochastic terms** strong impact on **short term prediction**

Lorenz 'butterfly effect' made it into a recent movie: 'An Inconvenient Truth' (2006) about former United States Vice President Al Gore. Directed by Davies Guggenheim

REFERENCES

Ghil, M. “Geophysical flows as dynamical system: the influence of Hide's experiment.” *Astronomy & Geophysis* 51 (2010) 4-28

Lorenz, E.N. “Deterministic Nonperiodic Flow” *J. Atmos. Sci. 2*0 (1963) 130-141

Lorenz, E.N. “Predictability – a problem partly solved.”, *Seminar on Predictability* (1996)

Lorenz, E.N. “The Essence of Chaos”, *University of Washington Press* (1993)

Orrel, D. “Model error and predictability over different timescales in the Lorenz'96 systems” *J. Atmos. Sci.* 60 (2003) 2219-2228